Electric charge: It is the fundamental property of matter due to which it produces and experiences electrical and magnetic effects.

There are two types of charges i.e +ve charge & -ve charge.

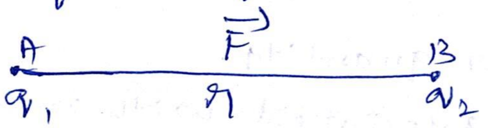
Eg:- The charge produced by rubbing glass rod with silk is +ve charge & by rubbing hard rubber with fur is -ve charge.

Properties of charge:-

1) Charge is a scalar quantity.
2) Charge is always associated with mass.
 a) when body gets +ve charge its mass decreases
 b) when body gets -ve charge its mass increases
3) Charge is quantised.
 $Q = \pm ne$ where n is integer
 $e = 1.6 \times 10^{-19} C$
4) Charge is conserved. i.e it can neither be created nor be destroyed, but only can be transferred from one body to another, without the charge being affected.
5) Like charges repel and unlike charges attract each other.
6) Accelerated charge radiates energy.
7) Charges given to a conductor always resides on its surface. In insulator it remains where it is placed.
8) Charge density of a irregular conducting body is not uniform. It is maximum where the radius of curvature is minimum.

Point charge: Any charge whose dimensions are much smaller than its distance from the point of observation is considered as a point charge. The concept of point charge is ideal.

Coulomb's law: It states that "the magnitude of the force between two point charges is directly ∝al to the product of the magnitude of the charges and inversly ∝al to the square of the distance between them."



$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = K \frac{q_1 q_2}{r^2}$$

K depends on nature of medium seperating the charges & the choice of the system of units for measurement of F, q & r.

In vector from $\vec{F} = K \frac{q_1 q_2}{r^2} \hat{r}$

$\hat{r}$ is the unit vector in the direction of the force.

In SI units, $K = 9 \times 10^{9}\ N m^{2} C^{-2}$

$K = \frac{1}{4\pi\varepsilon_0}$, $\varepsilon = \varepsilon_r \varepsilon_0$

ε_0 is ~~absolute~~ permittivity of the free space ~~or vaccum~~ air medium

$\varepsilon_0 = 8.8541 \times 10^{-12}\ C^{2} N^{-1} m^{-2}$ or $F m^{-1}$

ε_r - relative permittivity of the medium ($\varepsilon_r = 1$)

ε - absolute permittivity of the medium

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$\varepsilon_0 = \frac{1}{4\pi} \frac{1}{F} \frac{q_1 q_2}{r^2}$$

$$= C^{2} m^{-2} N^{-1}$$

$$\therefore \frac{1}{4\pi\varepsilon_0} = N m^{2} C^{-2}$$

Unit charge:- If $q_1 = q_2 = q$, $d = 1m$ & $F = 9 \times 10^9 N$ then

$$9 \times 10^9 = 9 \times 10^9 \frac{q^2}{1^2}$$

$$q^2 = 1 \text{ or } q = \pm 1$$

Thus unit charge may be defined as that quantity of charge which when placed at a distance of 1m away from an equal and similar charge in air or vacuum repels it with a force of $9 \times 10^9 N$

Limitations of coulomb's law:-

1) It is valid only for point charges
2) It is valid for charges at rest.
3) It is not applicable for charges in motion
4) It fails to explain the stability of nucleus.
5) It is not universal since it depends upon the medium in which the charges are placed.

~~Electric field strength~~ or

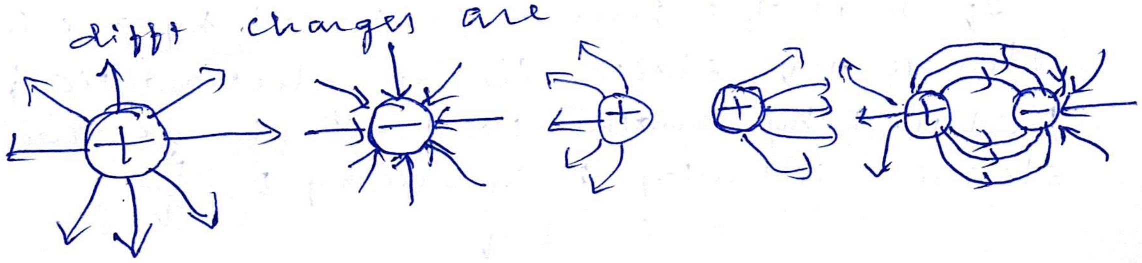
~~Intensity of electric field~~:-

Electric field;

The space surrounding a charge in which another charge experiences a force is called electric field.

The electric field is represented by the lines of force called Electric lines of force or lines of force.

The pattern of lines of force due to diff charges are



Characteristics of lines of force :-

1) Lines of force originate from +ve charge & terminate on -ve charge.
2) They never intersect.
3) They are straight for an isolated point charge.
4) They contract longitudinally. This accounts for the attraction betn 2 unlike charges.
5) They expand laterally. This accounts for the repulsion betn 2 like charges.

Electric Intensity or Electric field Strength (E) :-

Electric intensity at a point is defined as the electrostatic force acting on a unit +ve charge placed at that point. Its direction is same as that of the force.

Consider a point +ve charge +q placed at a point A. It produces an electrostatic field all around it. If a test charge $+q_0$ is placed at B it experiences a force F. The electric intensity at B is given by $E = \lim_{q_0 \to 0} \frac{F}{q_0}$

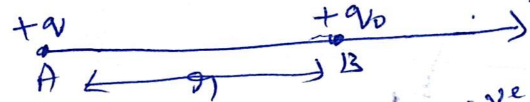
ie $E = \frac{F}{q_0}$

Force on charge in an electrostatic field :-

If F is the force experienced by a point charge +q placed in an electrostatic field produced by another point +ve charge then electric intensity is given by $E = \frac{F}{q}$

or $F = Eq$

Electric Intensity due to a point charge :-



$\vec{E} = \frac{F}{q_0}$

Consider a point +ve charge +q placed at a pt A. It produces an electrostatic field all around it. To find the electric ~~field~~ intensity at a pt B at a distance r from the pt A, a test charge q_0 is placed at B.

Force experienced by q_0 is

$$\bar{F} = \frac{1}{4\pi\varepsilon_0}\frac{q q_0}{r^2}$$

By defn of electric intensity,

$$E = \frac{F}{q_0} = \frac{1}{\cancel{q_0}}\frac{1}{4\pi\varepsilon_0}\frac{q \cancel{q_0}}{r^2}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}} \quad NC^{-1} \quad \frac{N \cancel{m^2}}{C^2}\,\frac{C}{\cancel{m^2}}$$

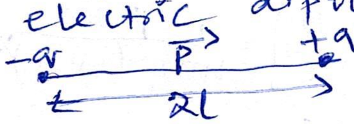
NOTE : 1) Electric intensity due to a pt +ve charge is always away from charge q_0.

point charge for -q.

It is towards -ve point charge.

Electric dipole :-

A pair of equal and opposite point charges seperated by a fixed distance is called an electric dipole. The product of the magnitude of either charge & the distance betn the charges is called electric dipole moment. Let the charges of dipole are -q & +q seperated by 2l m, then electric dipole moment p is given by $p = q \times 2l = 2ql$



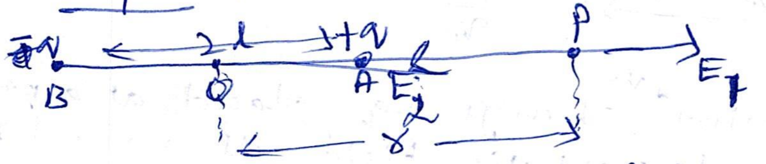
It is vector quantity

The SI unit is colomb-metre

Its direction is from -ve charge to +ve charge

Electric field due to an electric dipole

Electric field at a point on the axis of a dipole :-



The electric dipole is made up of two equal and opposite changes $+q$ and $-q$ coulomb seperated by a distance $2l$ metre as shown in fig. Let P be a point at distance r from the centre O of the dipole, at which the intensity of the field is to be determined. Let E_1 & E_2 be the intensities of electric field at P due to $+q$ & $-q$ resply.

$$\therefore E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{AP^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-l)^2}$$ in the dirn AP

$$\& \quad E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+l)^2}$$ in the dirn PB

The resultant intensity at P will be difference of the 2 intensities since they are in opposite directions. $E = E_1 - E_2$

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(r-l)^2} - \frac{q}{(r+l)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+l)^2 - (r-l)^2}{(r-l)^2(r+l)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{(\cancel{r^2}+\cancel{l^2}+2rl) - (\cancel{r^2}+\cancel{l^2}-2rl)}{(r^2-l^2)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{4rl}{(r^2-l^2)^2}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{2ql \times 2r}{(r^2-l^2)^2}\right] = \frac{1}{4\pi\varepsilon_0}\frac{p\,2r}{(r^2-l^2)^2}$$

Since $p = 2ql$

If $r \gg l$ then l^2 may be neglected.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p\cancel{r}}{r^{\cancel{4}\,3}}$$

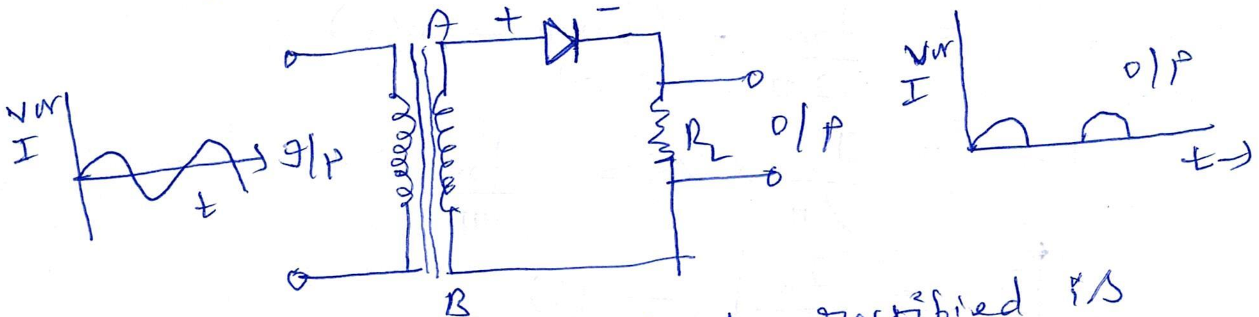
$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}}$$ N/C

$$N m^2/C^2 \cdot \frac{Cm}{m^3}$$

$$= \frac{Nm^2}{C^2} \cdot \frac{Cm}{m^3}$$

Rectifier: A Rectifier is an electronic circuit used for converting AC voltage or current into DC voltage or current. Basically Rectifiers are classified into
1) Half wave rectifier
2) Full wave rectifier.

Half wave rectifier: The circuit diagram is as shown below.



The AC voltage to be rectified is applied across the primary of the transformer and that of the secondary is available for rectification.
Let the voltage across the secondary of the transformer is

$$V_L = V_m \sin \omega t \quad \text{---} (1)$$

During the +ve half cycle, the diode is forward bias & hence it conducts. During -ve half cycle, the diode doesnot conduct. Thus in one half cycle of AC there is current thro' the load resistance R_L and we get an output voltage across R_L. There is negligibly small current in other half cycle and the output voltage practically zero. The voltage across R_L is is not a perfect dc but a pulsating dc and it is unidirectional.

Peak current: Assuming the resistance of the diode r_f (forward bias) & load resistance R_L, the peak current is $I_m = \dfrac{V_m}{R_L + R_f}$ — (2)

To find / The current waveform can be mathematically written as $I_L = I_m \sin wt$ for $0 \le wt \le \pi$ & $I_L = 0$ for $\pi \le wt \le 2\pi$

1) The average or DC value of the current, we find the net area of the current curve over one complete cycle & from $wt = 0$ to $wt = 2\pi$ and divide the area by the base 2π.

I_{av} or (I_{dc})

$$I_{dc} = \frac{1}{2\pi}\int_0^{2\pi} I_L \, d(wt)$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} I_m \sin wt \, d(wt) + \int_{\pi}^{2\pi} 0\right]$$

$$= \frac{1}{2\pi}(I_m)(-\cos wt)\Big|_0^{\pi}$$

$$= \frac{I_m}{2\pi} - (\underset{-1}{\cos\pi} - \underset{1}{\cos 0})$$

$$= \frac{I_m}{\cancel{2}\pi} \times \cancel{2} = \frac{I_m}{\pi}$$

$$\boxed{I_{dc} = \frac{I_m}{\pi}}$$ — (3)

and $V_{dc} = I_{dc} \times R_L$

$$= \frac{I_m}{\pi} \times R_L$$

$$= \frac{V_m}{(R_L + R_f)\pi} \times R_L$$

If $R_L >> R_f$, then

$$V_{dc} = \frac{V_m}{\cancel{R_L}\pi} \times \cancel{R_L}$$

$$\boxed{V_{dc} = \frac{V_m}{\pi}}$$ — (4)

2) Rms current and voltage :-

The value of rms current is given by $I_{rms} = \left[\frac{1}{2\pi}\int_0^{2\pi} I^2 \, d(\omega t)\right]^{1/2}$

$$= \left[\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2\omega t \, d(\omega t)\right]^{1/2} = \left(\frac{I_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right)d\omega t + \int_{\pi}^{2\pi} 0 \cdot d(\omega t)\right)^{1/2}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}} = \sqrt{\frac{I_m^2}{4\pi}\pi}$$

$$I_{rms} = \frac{I_m}{2} \quad \text{—— (5)}$$

and rms voltage across the load

$$V_{rms} = I_{rms} \times R_L$$

$$= \frac{I_m}{2} R_L$$

$$= \frac{V_m}{2(R_L + R_f)} R_L$$

If $R_L >> R_f$

$$V_{rms} = \frac{V_m}{2\cancel{R_L}} \cancel{R_L}$$

$$V_{rms} = \frac{V_m}{2} \quad \text{—— (6)}$$

3) Rectifier efficiency (η)

The rectifier efficiency tells us that what percentage of total input ac power is converted into useful dc power output. It is defined as the ratio of dc output power to the ac ~~output~~ input power.

$$\text{i.e. } \eta = \frac{P_{dc}}{P_{ac}} = \frac{\text{dc o/p power}}{\text{ac i/p power}} \times 100\%$$

DC power delivered to the load $= I_{dc}^2 R_L$

$$P_{dc} = \frac{I_m^2}{\pi^2} R_L$$

Ac power supplied from the secondary of the transformer

$$P_{ac} = I_{rms}^2 (R_f + R_L)$$
$$= \frac{I_m^2}{4} (R_f + R_L)$$

$$\eta = \frac{\frac{I_m^2 R_L}{\pi^2}}{\frac{I_m^2}{4}(R_f + R_L)} \times 100\%$$

$$= \frac{4}{\pi^2} \frac{1}{\left(\frac{R_f + R_L}{R_L}\right)} = \frac{4}{\pi^2} \frac{1 \times 100}{\left(1 + \frac{R_f}{R_L}\right)}$$

$$= \frac{0.4057}{1 + \frac{R_f}{R_L}} = \frac{40.57}{\left(1 + \frac{r_f}{R_L}\right)} \%$$

Theoritically the maximum value of rectifier efficiency of a halfwave rectifier is 40.6 % when $\frac{R_f}{R_L} = 0$ if $R_L >> R_f$

Ripple factor (γ) : It is defined as the ratio of the rms value of ac component of current or voltage to the dc value of current or voltage.

Ac fluctuation ie $\gamma = \frac{I_{ac} = I'_{rms}}{I_{dc}}$

is $I' = I_L - I_{dc}$

$$\gamma = \frac{I'_{rms}}{I_{dc}}$$

$$= \frac{\sqrt{I_{rms}^2 - I_{dc}^2}}{I_{dc}}$$

$$= \sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1}$$

But $I_{rms} = \frac{I_m}{2}$

$I_{dc} = \frac{I_m}{\pi}$

The rms value of ac fluctuation is

$$I'_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (I_L - I_{dc})^2 \, d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (I_L^2 + I_{dc}^2 - 2 I_L I_{dc}) \, d(\omega t)}$$

Since $\frac{1}{2\pi}\int_0^{2\pi} I_L^2 \, d(\omega t) = I_{rms}^2$

$\frac{1}{2\pi}\int_0^{2\pi} I_{dc}^2 \, d(\omega t) = I_{dc}^2$

$\frac{1}{2\pi}\int_0^{2\pi} 2 I_L I_{dc} \, d(\omega t) = 2 I_{dc} \times \frac{1}{2\pi}\int_0^{2\pi} I_L \, d(\omega t)$

$= 2 I_{dc} I_{dc} = 2 I_{dc}^2$

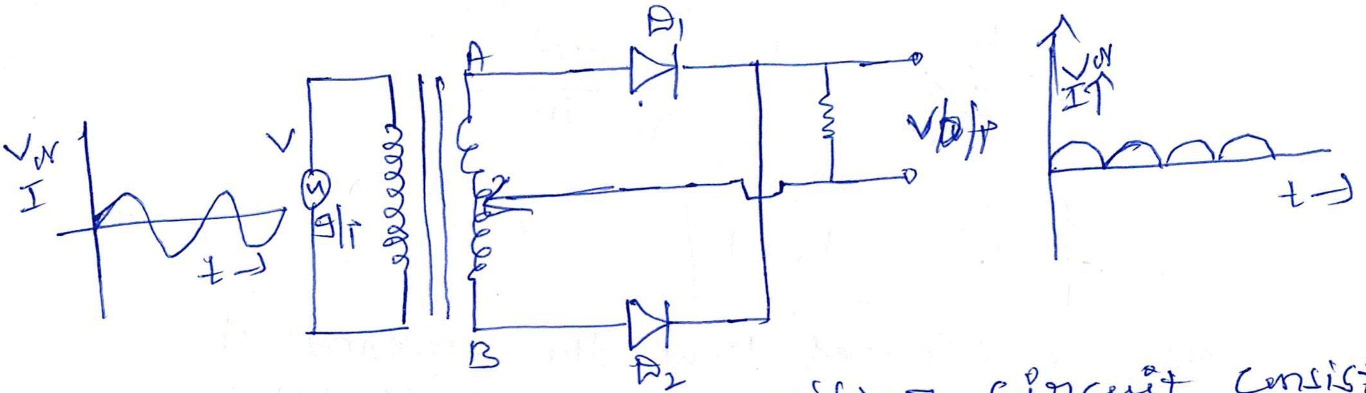
$\therefore I'_{rms} = \sqrt{I_{rms}^2 - I_{dc}^2}$

$$= \sqrt{\frac{\frac{I_{m}^{2}}{4}}{\frac{I_{m}^{2}}{\pi^{2}}} - 1} = \sqrt{\frac{\pi^{2}}{4} - 1}$$

$$\boxed{\gamma = 1.21}$$

It indicates that the amount of ac present in the output is 121% of dc voltage. Thus the half wave rectifier is not very successful in converting ac into dc.

Full wave Rectifier:



A full wave rectifier circuit consist of two diods D_1 and D_2 connected to the secondary of the stepdown transformer. The input AC signal is fed to the primary of the transformer. During the +ve half cycle end A becomes +ve and B becomes -ve, so the diode D_1 is forward biased and D_2 is reverse biased. As a result of this the diode D_1 conducts where D_2 donot conducts. Thus O/P appears across the load resistance.

During the -ve half cycle end A becomes -ve & B is +ve. Thus diode D_1 is reverse biased & D_2 is forward biased. ∴ during -ve half cycle diode D_2 will conduct the current and D_1 will not conduct. Thus output appears across the load resistance. During both the half cycles current flows thro' the load in the same direction. The output voltage is developed across the load R_L during the entire cycle. In this way full wave rectifier converts complete input ac signal to output dc signal.

Let the current through the rectifier circuit $I = I_m \sin \omega t$

$$I_m = \frac{V_m}{R_f + R_L}$$

R_f - forward resistance of diode
R_L - load resistance.

Average value of current & voltage:-

1) $$I_{dc} = \frac{1}{2\pi}\int_0^{2\pi} I \, d(\omega t)$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} I \, d(\omega t) + \int_{\pi}^{2\pi} -I \, d(\omega t)\right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} I_m \sin \omega t \, d(\omega t) + \int_{\pi}^{2\pi} -I_m \sin \omega t \, d(\omega t)\right]$$

$$= \frac{1}{2\pi}\left[-I_m \cos \omega t \Big|_0^{\pi} + I_m \cos \omega t \Big|_{\pi}^{2\pi}\right]$$

$$= \frac{1}{2\pi}\left[-I_m(-1-1) + I_m(1+1)\right]$$

$$= \frac{1}{2\pi}\left[2I_m + 2I_m\right] = \frac{\cancel{4}^{2} I_m}{\cancel{2}\pi}$$

$$\therefore \boxed{I_{dc} = \frac{2I_m}{\pi}}$$

$$V_{dc} = I_{dc} \times R_L$$

$$= \frac{2I_m}{\pi} \times R_L$$

$$= \frac{2}{\pi} \frac{V_m}{R_f + R_L} \times R_L$$

If $R_L >> R_f$, R_f can be neglected.

$$\therefore V_{dc} = \frac{2}{\pi} \frac{V_m}{\cancel{R_L}} \times \cancel{R_L}$$

$$\boxed{V_{dc} = \frac{2V_m}{\pi}}$$

2) I_{rms} and V_{rms}:

$$I_{rms} = \left[\frac{1}{2\pi}\int_0^{2\pi} I^2 \, d(\omega t)\right]^{1/2}$$

$$= \left\{\frac{1}{2\pi}\left[\int_0^{\pi} I^2 d(\omega t) + \int_{\pi}^{2\pi} (-I)^2 \cdot d(\omega t)\right]\right\}^{1/2}$$

$$= \left\{\frac{1}{2\pi}\left[\int_0^{\pi} I_m^2 \sin^2 \omega t \, d(\omega t) + \int_{\pi}^{2\pi} I_m^2 \sin^2 \omega t \, d(\omega t)\right]\right\}^{1/2}$$

$$= \left\{\frac{1}{2\pi}\left[\int_0^{\pi} \frac{I_m^2}{2}(1-\cos 2\omega t)\, d\omega t + \int_{\pi}^{2\pi} \frac{I_m^2}{2}(1-\cos 2\omega t)\, d\omega t\right]\right\}^{1/2}$$

$$= \left\{\frac{1}{2\pi}\left[\frac{I_m^2}{2}\pi + \frac{I_m^2}{2}\pi\right]\right\}^{1/2}$$

$$= \left(\frac{1}{2\pi} \frac{\cancel{2} I_m^2 \cancel{\pi}}{\cancel{2}}\right)^{1/2} = \left(\frac{I_m^2}{2}\right)^{1/2}$$

$$\therefore \boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$$

$$V_{rms} = I_{rms} \times R_L = \frac{I_m}{\sqrt{2}} R_L$$

$$= \frac{V_m}{\sqrt{2}(R_f + R_L)} R_L$$

If $R_L \gg R_f$, R_f can be neglected

$$\therefore \boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$$

Efficiency of Full wave rectifier (η):-

$$\eta = \frac{P_{dc}}{P_{ac}}$$

$$P_{dc} = I_{dc}^2 R_L$$

$$= \left(\frac{2I_m}{\pi}\right)^2 R_L$$

$$= \frac{4I_m^2}{\pi^2} R_L$$

$$P_{ac} = I_{rms}^2 (R_f + R_L)$$

$$= \left(\frac{I_m}{\sqrt{2}}\right)^2 (R_f + R_L)$$

$$\eta = \frac{\frac{4\cancel{I_m^2}}{\pi^2} \times R_L}{\frac{\cancel{I_m^2}(R_f + R_L)}{2}}$$

$$= \frac{8}{\pi^2} \frac{1}{\left(\frac{R_f + R_L}{R_L}\right)} = \frac{0.81}{\left(1 + \frac{R_f}{R_L}\right)}$$

Thus the efficiency of a full wave rectifier is double that of a half wave rectifier under identical conditions. The maximum possible efficiency of a full wave rectifier is 81% when $R_f << R_L$.

Ripple factor (r) : It is defined as the ratio of the rms value of the ac component to the average or dc value of load current.

$$r = \frac{I_{ac}}{I_{dc}} = \frac{I'_{rms}}{I_{dc}}$$

After simplification $r = \sqrt{\frac{I_{rms}^2}{I_{dc}^2} - 1}$

$$= \sqrt{\frac{I_m^2/2}{4I_m^2/\pi^2} - 1} = \sqrt{\frac{\pi^2}{8} - 1}$$

$$= 0.482$$

It shows that, the ripple contents in the output are 48.2% of the dc components. It is much less than that of the half-wave rectifier.

Advantages : (1) It uses both the cycles

2) Its ripple factor ($r = 48.2\%$) is low. That means o/p voltage contains less no. of ac components.

3) Its efficiency ($\eta = 81.2\%$) is double that of half wave rectifier.

Cathode Ray oscilloscope (CRO):

It is a device used to observe visual representation of the waveform of alternating voltages and other oscillating effect of high frequency.

It was developed by Thomson.

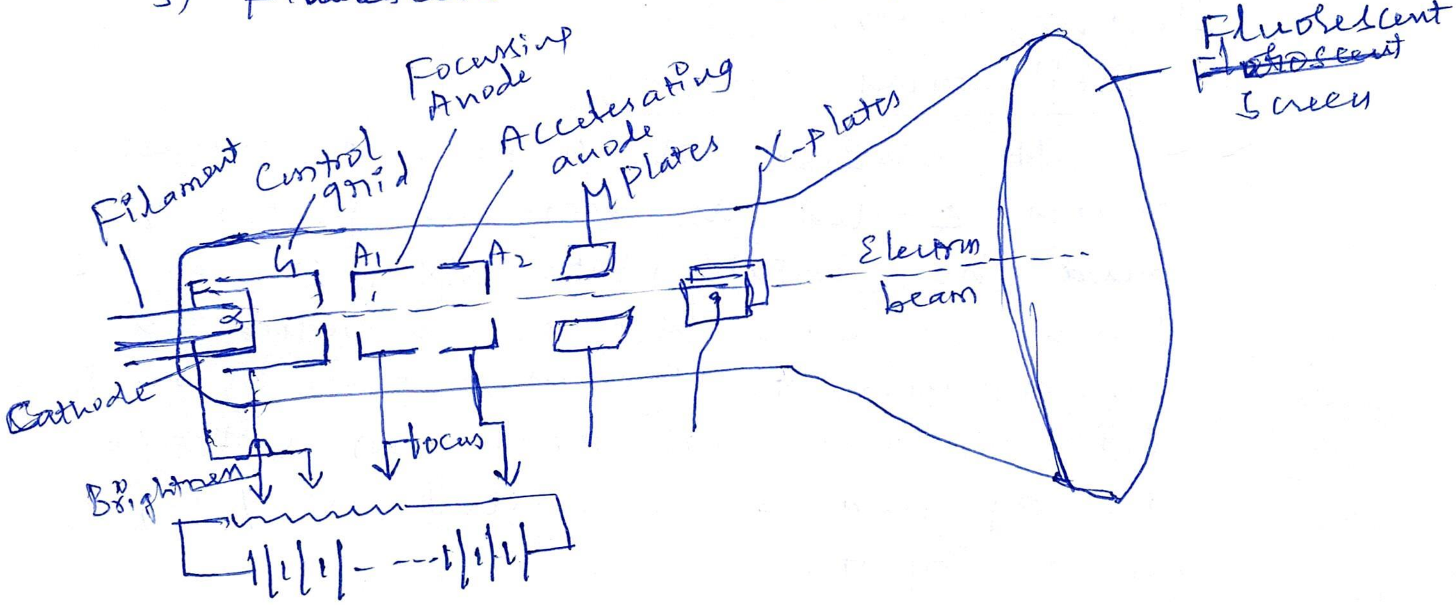
Principle: It works on the principle that an electronic beam is made to deflect under the simultaneous effect of electric and magnetic fields and the bright spot is produced on the fluorescent screen.

The main constituents of a cathode ray oscilloscope are

1) cathode ray tube
2) power supply system
3) Time base circuit

Cathode ray tube:- It is the heart of the CRO. It consists the following parts.

1) Electron gun (2) deflecting system
3) Fluorescent screen (4) Glass tube (5) Base.



1) Electron gun :- It contains the filament F (ie cathode) which is heated by a battery. The filament produces the electrons. The filament is surrounded by control grid G which is a nickel cylinder provided with a small, central hole co-axial with the axis of the tube. The grid is kept at a negative potential wrt cathode. The intensity of the luminous spot can be controlled by varying grid potential.

After leaving the grid, the electronic beam passes thro' two anodes A_1 and A_2. A_1 is called focussing anode and A_2 is called accelerating anodes. Both are kept at +ve potential with respect to the grid. The +ve potential of A_1 is of the order of 350-700 volt and that of A_2 is of about 2000 volts. Both the anodes help to focus and accelerate the beam. The whole unit is called Electron gun.

2) Deflecting system:

It consists of 2 pairs of parallel plates called as vertical plates YY and horizontal plates XX. one of the plates in each set is connected to the ground. To the other plate of each set, the external deflection voltage is applied & is called as Y input or X-input.

If the +ve voltage is applied to the Y input (V_y), the electron beam gets deflected vertically upwards, if -ve voltage is applied, then it gets deflected downwards.

Similarly if the +ve voltage is applied to X input (V_x), the electron beam gets deflected towards that plate & if -ve voltage is applied to V_x, the electron beam gets deflected away from that X-plate.

If the voltages are applied simultaneously to both vertical and horizontal deflecting plates, we get the Lissajous figures on the screen.

The whole unit is enclosed in a glass bulb which is evacuated completely.

3) <u>Fluorescent screen</u>:-

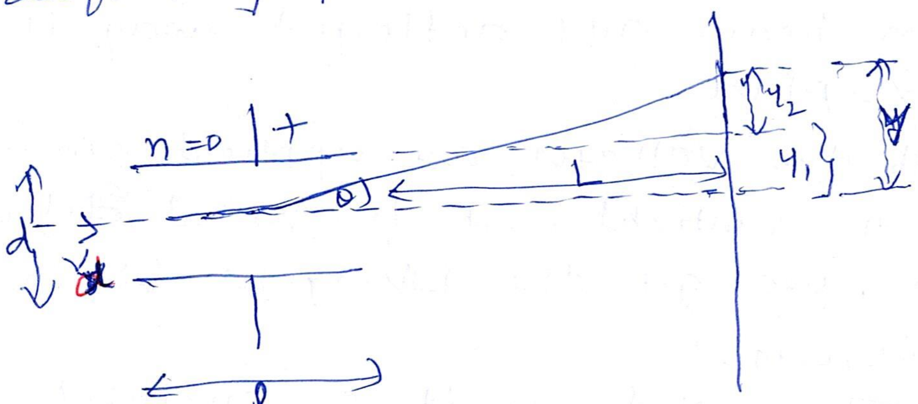
The front inner portion of the cathode ray tube is coated with a fluorescent material such as phosphor which emits light when bombarded with electrons

4) <u>Glass tube</u>: All the components of the CRO are enclosed in an evacuated glass tube called envelope which allows the emitted electrons to move about freely from one end to the other end of the tube.

Base : The CRO is fixed on the base through which the connections are made to various parts.

Deflection sensitivity for electrostatic field :-

Deflection sensitivity is defined as the amount of deflection of the electron spot produced on the screen when a voltage of one volt dc is applied betn corresponding deflecting plates.



l - length of each plate

d - distance betn the plates

L - distance betn edge of the plate to the screen.

V_d - potential applied betn the plates

V_a - accelerating potential applied to the last anode.

e - charge of electron

m - mass of electron

$y = y_1 + y_2$ - total deflection.

From fig, we have

$$\tan\theta = \frac{y}{L + \frac{l}{2}} \quad \text{--- (1)}$$

It can be shown that, the motion of a charged particle in an uniform transverse electric field is given by

$$\tan\theta = \frac{e E_d}{m v_x^2} l \quad -② $$

where V_x is the velocity with which the electron enters the deflecting plates E_d is the deflecting field.

① = ②

$$\frac{Y}{L+\frac{l}{2}} = \frac{e E_d}{m v_x^2} l$$

$$\frac{Y}{D} = \frac{e E_d}{m v_x^2} l \quad \text{where } D = L + \frac{l}{2}$$

If V_d be the p.d betn the plates then

$$E_d = \frac{V_d}{d}$$

$$\therefore \frac{Y}{D} = \frac{e V_d}{d\, m v_x^2} l.$$

Deflection sensitivity = $\frac{\text{Deflection}}{\text{Deflecting voltage}}$

$$\text{ie } \frac{Y}{V_d} = \frac{e D l}{m v_x^2 d} \quad -③$$

If the electron beam before entering deflecting plates is accelerated to achieve this velocity V_x by accelerating voltage V_a, then $\frac{1}{2} m v_x^2 = e V_a$

$$m v_x^2 = 2 e V_a \quad -④$$

Subs (4) in (3) we have

$$\frac{y}{V_d} = \frac{\cancel{e} D l}{2 \cancel{e} V_a d}$$

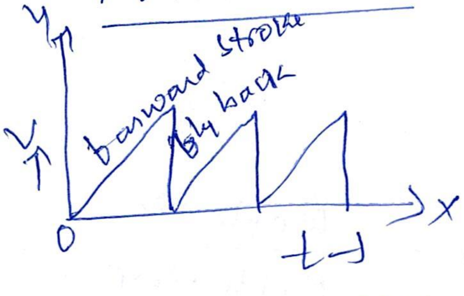
∴ The deflection sensitivity is given by

$$= \frac{l}{2 d V_a} D \quad \text{where } D = L + \frac{l}{2}$$

Thus the deflection sensitivity is

1) directly $\propto$al to the length of deflecting plates
2) directly $\propto$al to $D = L + \frac{l}{2}$
3) Inversely $\propto$al to final accelerating voltage (V_a)
4) Inversely $\propto$al to the spacing d betn. the plates.

Time base



A time base is a scanning generator is a circuit which generates a saw tooth wave form and makes the spot to deflect in a horizontal or vertical direction linearly with time. So it provides a regularly traced time axis on the fluorescent screen during the forward strokes which are repeated successively.

Need of a Time base voltage :-

If a dc voltage is applied across the X-plates, the spot of light will move along a horizontal line on the screen. Similarly if an ac voltage alone is applied to Y-plates, the spot of light would move vertically up and down on the screen. It does not reveal the actual nature of the waveform. To get the actual waveform, a linear time base voltage is applied across the X-plates. It is used to generate a voltage which rises linearly with a certain time to a certain value and then drops suddenly to zero & the cycle is repeated. Under this voltage, the spot sweeps linearly across the screen from left to right & then flies back quickly to the starting position for the next sweep. This horizontal sweeps, appears stationary due to persistence of vision and hence provide the time base to oscilloscope.

When the vertical motion of the spot produced by Y-plates due to alternating voltage V is superimposed on the horizontal sweep produced by X-plate, the actual nature of the wave form is obtained on the screen.

Uses of cathode ray oscilloscope :-

1) Measurement of direct or alternating voltage :-

Deflection sensitivity is defined as the amount of deflection of the electron spot produced when a voltage of one volt is applied betn the corresponding deflection plates.

When dc voltage is applied across the plates the deflection is observed on the screen. If this deflection is multiplied by the deflection sensitivity, we get the magnitude of the applied dc voltage.

2) Measurement of alternating voltage :-

The alternating voltage of sinusoidal waveform is applied across the Y plates, a ~~st line~~ sinusoidal waveform is obtained on the screen. If the length of this peak is multiplied by the deflection sensitivity we get the peak to peak ac voltage. Dividing this by $\sqrt{2}$, we get rms value of ac voltage.

3) Measurement of frequency :-

The control knobs are adjusted [time / div control] so that signal appears on the screen. The length of one complete wave is noted on the horizontal graduated scale & is measured as period. The reciprocal of this gives frequency.

Lissajous figures :

The ac signal whose frequency is to be measured is applied across the vertical input ie Y plates. Instead of a linear time base, a standard oscillator is applied across X plates. The gains of vertical and horizontal amplitudes are adjusted to have the same instantaneous amplitudes. Now the electron beam moves vertically and horizontally under the forces ∝al to the instantaneous values of 2 voltages. As a result, Lissajous figures are seen on the screen.

Filters

An electronic device which removes or minimise the ac components from the rectifier output is known as filter.

Generally inductors and capacitors are used for filtering because of their opposite frequency characteristics.

Connection of Inductor & capacitor in the circuit:-

We know that $X_L = 2\pi f L$

for dc, $f = 0$ $\therefore X_L = 0$

Thus inductor acts as a short ckt for dc ie it allows dc components so it should be connected in series with the load.

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ For dc $f = 0$, $X_C = \infty$

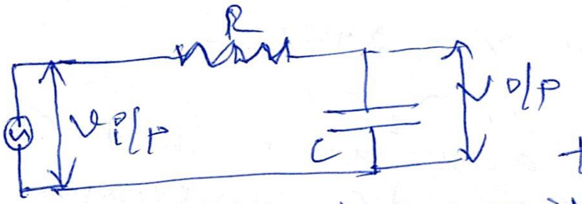
Thus the capacitor acts a open ckt for dc. It blocks entire dc components. ie capacitor cannot be connected in series with the load, it always be connected in parallel with the load.

There are four general type of filter circuits.

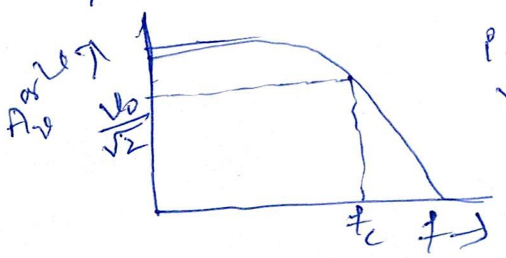
1) Low pass filter
2) High pass filter
3) Band pass filter
4) Band stop filter.

<u>Low pass filter</u> :- It is a circuit which passes low frequencies but stops higher frequencies. A low pass filter is so designed that it transmits currents of all frequencies ~~(f_c)~~ from zero to certain cutoff frequencies (f_c) without attenuation. All frequencies higher than f_c are attenuated.

A simple low pass RC filter is as shown in fig.



The output voltage is measured across the capacitor which offers fixed opposition to all frequencies whereas the reactance of the capacitor decreases with increase of frequency. Thus the circuit passes low frequencies readily. As the frequency increases the reactance of the capacitor decreases. At very high frequencies the capacitor acts as a virtual short circuit and the output falls to zero. The frequency response of RC low pass filter is as shown below



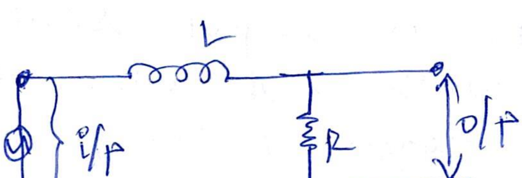
At f_c the series current I is at 70.7% of its maximum value. The formula for cutoff frequency can be calculated as follows

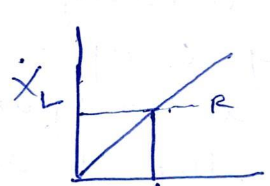
At f_c, $R = X_c$

$$R = \frac{1}{\omega C} = \frac{1}{2\pi f_c C}$$

$$\boxed{\therefore f_c = \frac{1}{2\pi R C}}$$

Low pass RL filter





In this case the Inductor coil offers high resistance to high frequencies and allows low frequencies to pass thro' it. Hence low frequencies upto f_c pass through the coil.

At f_c, $R = X_L$

$$R = \omega L$$

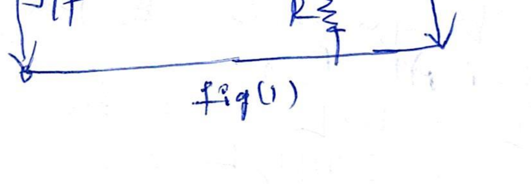
$$R = 2\pi f_c L$$

$$\therefore \boxed{f_c = \frac{R}{2\pi L}}$$

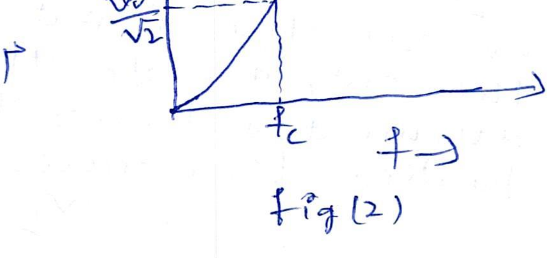
High pass filter:

It is a circuit which passes high frequencies but stops low ones. A high pass circuit is so designed that transmits currents of all frequencies lying between infinity and certain cut off frequency f_c. All other frequencies below f_c are attenuated.

High pass RC filters: circuit is as shown below.



fig(1)



fig(2)

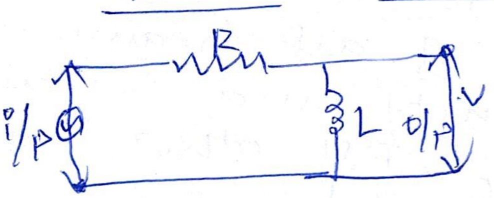
Since the reactance of capacitor decreases with increasing frequency, the higher frequency components in input signal appear at output with less attenuation than to the lower frequency components. At very high frequencies the reactance X_C is so small that the capacitor acts almost as a short circuit and virtually all the input appears at the output.

The frequency response of high pass filter is as in fig (2).

$$\text{At } f_c, \quad R = X_C = \frac{1}{\omega C} = \frac{1}{2\pi f_c C}$$

$$\therefore \boxed{f_c = \frac{1}{2\pi R C}}$$

High pass RL filter :



For very low frequencies X_L is very small the L appears as a short. Since the voltage across a short is zero, the output voltage for low frequencies must be zero. On the other hand for higher frequencies X_L is very high and L acts as open circuit. $\therefore$ all the input voltage appears across the output. In this way high pass filter attenuates lower frequencies & passes higher frequencies. At f_c,

$$R = X_L = \omega L = 2\pi f_c L$$

$$\therefore \boxed{f_c = \frac{R}{2\pi L}}$$

Band pass filters;

It transmitts a specified Band pass frequencies and stops others.

In otherwards it transmitts currents of all frequencies whose values lies between the cut off frequencies f_{c_1} & f_{c_2}. It stops all frequencies lying either below f_{c_1} or above f_{c_2}.

Band pass filter is a combination of high pass and low pass filter. The circuit is as shown fig ①.

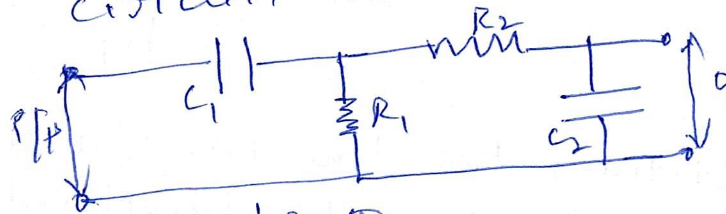


fig ①

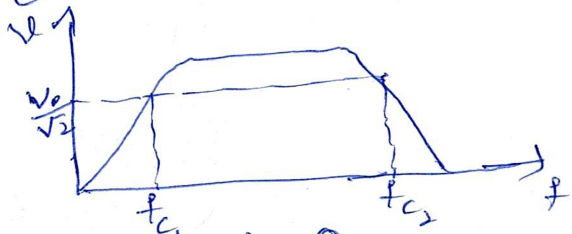


fig ②

R_1C_1 constitutes the high pass filter while R_2C_2 constitutes the low pass filter. The cut off frequency of the high pass filter is f_{c_1} and that of low pass filter is f_{c_2}. Frequency response is as shown in fig (2). The cut off frequencies is given by

$$f_{c_1} = \frac{1}{2\pi R_1 C_1}$$

and $$f_{c_2} = \frac{1}{2\pi R_2 C_2}$$

Band Stop filter :

It passes all frequencies except a specified band. It is also known as band elimination filter. This filter is a combination of high pass and low pass filter. The circuit is as in fig 1

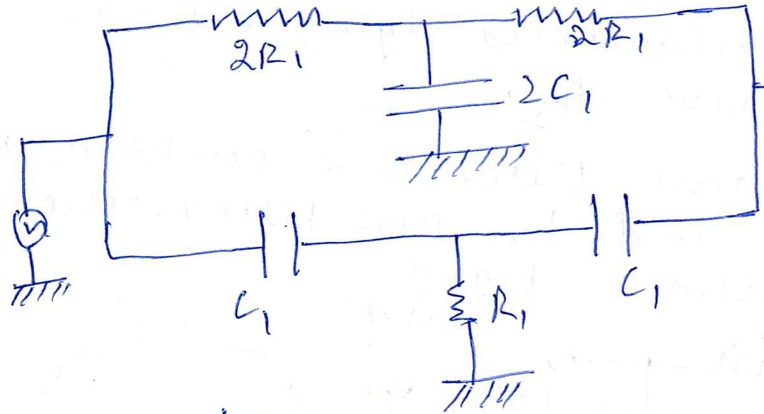


fig 1

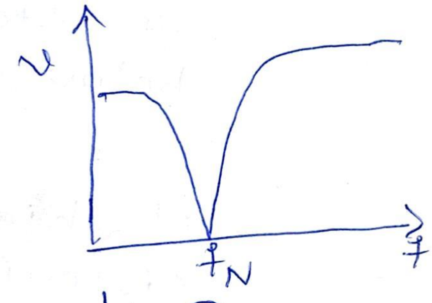


fig 2

$2R_1$ & $2C_1$ constitutes the low pass filter section while the components identified as R_1, C_1 constitute the high pass filter section. The individual filters are parallel with each other. The frequency of maximum attenuation is called notch frequency & is given by $f_N = \frac{1}{4\pi C_1 R_1}$

The frequency response of band stop filter is as in fig 2.

From equs (1) & (2), It is found that the current leads the applied emf by $\frac{\pi}{2}$

X_C, the capacitive reactance is the ratio of rms value of voltage across the capacitor to the rms value of current.

$$X_C = \frac{E_{rms}}{I_{rms}}$$

Behaviour of capacitor in AC & dC

1) In case of DC;

DC has no frequency i.e $f = 0$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{0} = \infty$$

capacitor offers infinite ~~resistance~~ opposition to the flow of DC.

$\therefore$ dc does not flow through the capacitor.

2) In the case of AC;-

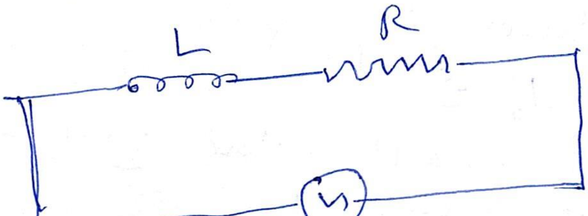
$$X_C \propto \frac{1}{f}$$

capacitive reactance is inversly $\propto$al to the frequency of applied voltage.

condenser offers negligible reactance at high frequency and works as by pass capacitor

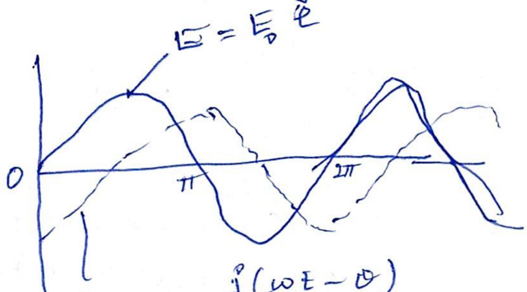
condenser offers high reactance at low frequency and works as blocking condensor.

AC circuit containing Inductance and Resistance in series :-



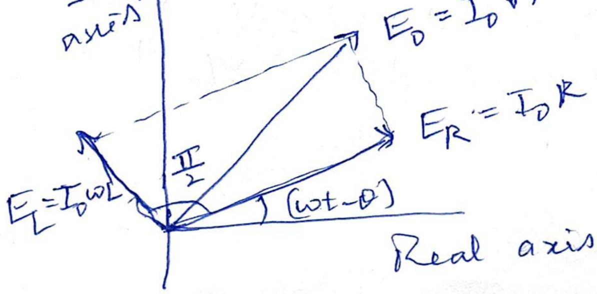
$E = E_0 \sin\omega t$

$E = E_0 e^{j\omega t}$



$I = I_0 e^{j(\omega t - \theta)}$

Imaginary axis



Real axis

Consider an ac circuit containing the inductance and resistance in series with an alternate emf $E = E_0 e^{j\omega t}$ — (1)

R is the impedence due to resistance R

$j\omega L$ is the impedence due to inductance L

Total vector impedence, $Z = R + j\omega L$

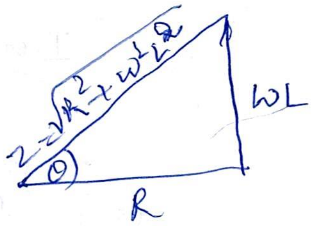
The ~~inst~~ instantaneous current is given by $I = \frac{\text{applied emf}}{\text{Impedence}} = \frac{E_0 e^{j\omega t}}{R + j\omega L}$ — (2)

$\frac{1}{R+j\omega L}$ can be written as

$$\frac{1}{R+j\omega L} = \frac{R - j\omega L}{(R+j\omega L)(R-j\omega L)}$$

$$= \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

$$= \frac{R}{R^2+\omega^2 L^2} - j\frac{\omega L}{R^2+\omega^2 L^2}$$



$\alpha = \frac{R}{R^2+\omega^2 L^2}$ and $\beta = \frac{\omega L}{R^2+\omega^2 L^2}$

ie $\frac{1}{R+j\omega L} = \alpha - j\beta$

From fig $\tan\theta = \frac{\omega L}{R}$, $\cos\theta = \frac{R}{\sqrt{R^2+\omega^2 L^2}}$ $\sin\theta = \frac{\omega L}{\sqrt{R^2+\omega^2 L^2}}$

ie $\frac{1}{R+j\omega L} = \alpha - j\beta = \frac{1}{\sqrt{R^2+\omega^2 L^2}}(\cos\theta - j\sin\theta)$

$$= \frac{1}{\sqrt{R^2+\omega^2 L^2}} e^{-j\theta}$$

equ (1) reduces to

$$I = I_0 e^{j(\omega t - \theta)} \quad \text{— (3)}$$

Comparing eqns (1) & (3), the emf leads the current by an angle θ.

The voltage E_R across the resistance is in phase with the current while voltage E_L across the inductance leading the current I by an angle $\frac{\pi}{2}$.

Special cases :-

1) If ~~the~~ ωL is too large compared to R, then R may be neglected.

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

$$Z = \omega L = 2\pi f L$$

It happens when the frequency is very high.

Now the current becomes

$$I = I_0 e^{j(\omega t - \theta)}$$

$$I_0 = \frac{E_0}{\omega L} = \frac{E_0}{2\pi f L}$$

For a given value of L, $I_0 \propto \frac{1}{f}$

2) The phase angle $\theta = \tan^{-1} \frac{\omega L}{R}$ increases along with f.

Thus the current and emf can be brought into phase by introduction of non-inductive resistance. Practically this is adopted in AC voltmeters and wattmeters.

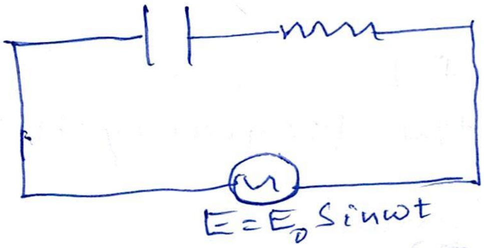
3) If the resistance R is too large compared to the inductive resistance, then ωL may be neglected.

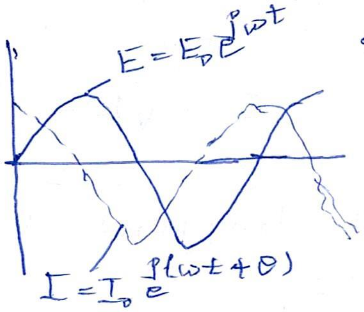
$$Z = \sqrt{R^2 + \omega^2 L^2} = R \text{ and } \theta \to 0$$

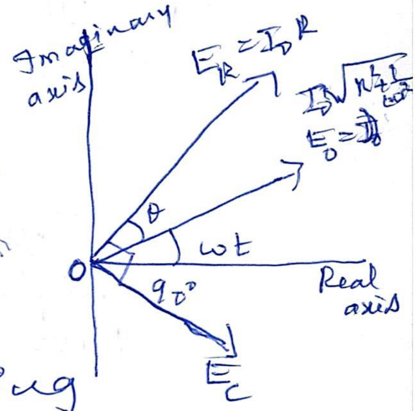
$$I = I_0 e^{j\omega t} = \frac{E_0 e^{j\omega t}}{R}$$

$$\text{ie } I = \frac{E}{R}$$

AC Circuit containing Resistance & Capacitance in series :-







Consider an a.c circuit containing a capacitance C and a resistance R connected in series with an emf.

$$E = E_0 e^{j\omega t} \quad \text{---(1)}$$

R is the impedence due to resistance R

$\frac{1}{j\omega C}$ is the impedence due to capacitance C.

Instantaneous current $I = \dfrac{\text{applied emf}}{\text{Total impedence}}$

$$I = \frac{E_0 e^{j\omega t}}{R + \frac{1}{j\omega C}} = \frac{E_0 e^{j\omega t}}{R - \frac{j}{\omega C}} \quad \text{---(2)}$$

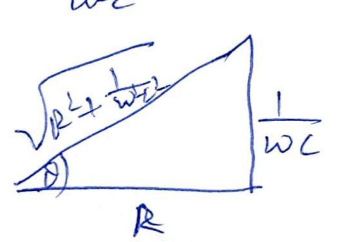
$\dfrac{1}{R - \frac{j}{\omega C}}$ can be written as

$$\frac{1}{R - \frac{j}{\omega C}} = \frac{R + \frac{j}{\omega C}}{\left(R - \frac{j}{\omega C}\right)\left(R + \frac{j}{\omega C}\right)} = \frac{R + \frac{j}{\omega C}}{R^2 + \frac{1}{\omega^2 C^2}}$$

$$= \frac{R}{R^2 + \frac{1}{\omega^2 C^2}} + \frac{j/\omega C}{R^2 + \frac{1}{\omega^2 C^2}}$$

Put $\alpha = \dfrac{R}{R^2 + \frac{1}{\omega^2 C^2}}$ and $\beta = \dfrac{1/\omega C}{R^2 + \frac{1}{\omega^2 C^2}}$

$$\tan\theta = \frac{\frac{1}{\omega C}}{R} = \frac{1}{\omega C R}$$



$$\frac{1}{R - \frac{j}{\omega C}} = \alpha + j\beta = \frac{1}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}(\cos\theta + j\sin\theta)$$

$$= \frac{1}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} e^{j\theta}$$

eqn. (2) reduces to $I = \dfrac{E_0 e^{j(\omega t+\theta)}}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$ ---

$I = I_0 e^{j(\omega t+\theta)}$ ---(3) where

$I_0 = \dfrac{E_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$ The current leads the applied voltage by θ.

The voltage E_R across the resistance is in phase with current, while the voltage E_C across condenser is lagging the current I by an angle $\frac{\pi}{2}$.

Special cases :-

1) We know that $I_0 = \frac{E_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$

If $C \to 0$, $I_0 \to 0$

It means that with zero capacity, no current flows thro' the circuit. This causes a break in the ckt. The same result will be obtained with low frequencies.

2) If $C \to \infty$ then $I_0 = \frac{E_0}{R}$. Thus a capacitor of infinite capacity does not offer any resistance to AC. The same result follows if the frequency is very high.

3) If $R \to 0$, then $I_0 = E_0 \omega C$ and $\tan\theta \to \infty$ or $\theta = \frac{\pi}{2}$. Thus the current is leading by $\frac{\pi}{2}$ in phase wrt emf. (11a)

LCR series circuit:

Consider an ac circuit containing inductance (L), capacitance (C) and resistance (R) joined in series with an applied emf $E = E_0 e^{j\omega t}$.

$E = E_0 e^{j\omega t}$

Let Z_R, Z_L and Z_C be the resistance, inductive reactance and capacitive reactances respectively.

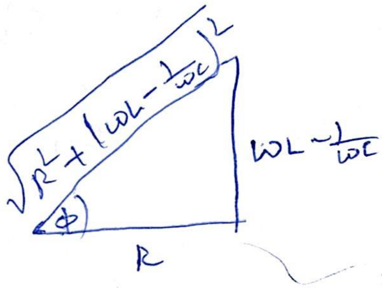
The total impedence is

$$Z = Z_R + Z_L + Z_C$$

$$= R + j\omega L - \frac{j}{\omega C} \quad \text{since } Z_C \text{ is -ve}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$I = \frac{E_0 e^{j\omega t}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$



$$\frac{1}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{R - j\left(\omega L - \frac{1}{\omega C}\right)}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \alpha - j\beta$$

where $\alpha = \frac{R}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$ and $\beta = \frac{\omega L - \frac{1}{\omega C}}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$$\tan\phi = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R} \qquad \cos\phi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\sin\phi = \frac{\omega L - \frac{1}{\omega C}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\therefore \frac{1}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}(\cos\phi - j\sin\phi)$$

$$= \frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} e^{-j\phi}$$

$$I = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} e^{j(\omega t - \phi)} \quad \text{or} \quad I = I_0 e^{j(\omega t - \phi)}$$

Electrical Resonance :

As the frequency of AC applied to LCR circuit increases, ωL increases whereas $\frac{1}{\omega C}$ decreases. At the particular value of frequency f_0, $\omega L = \frac{1}{\omega C}$. This frequency is known as the resonant frequency. Now the current becomes maximum.

The phenomenon in which inductive reactance becomes equal to the capacitive reactance, so that the current in the LCR circuit becomes maximum is called electrical resonance.

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$2\pi f_0 = \frac{1}{\sqrt{LC}} \quad \text{or} \quad \boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

f_0 is resonant frequency.

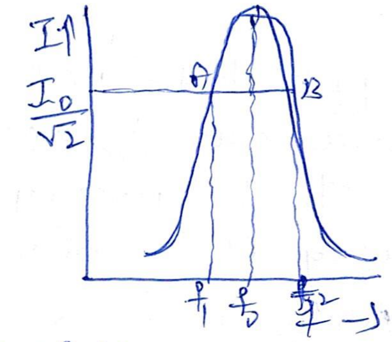
Acceptor Circuit: At the resonance, impedence of the LCR circuit is minimum. The circuit accepts or admits maximum current whose frequency is equal to the natural frequency of the circuit and suppresses the currents of other frequencies. Such circuit is called the Acceptor.

NOTE: 1) If $\omega L > \frac{1}{\omega C}$, the phase difference is +ve, the emf leads current by ϕ. The ckt behaves as inductive ckt.

2) If $\omega L < \frac{1}{\omega C}$, the phase difference is -ve now emf lags behind the current by ϕ. The ckt behaves as capacitive ckt.

3) If $\omega L = \frac{1}{\omega C}$, the phase difference is zero. Now the current and emf are in the same phase, the ckt behaves as resistive.

Band width :-



Let I_0 falls to $\frac{I_0}{\sqrt{2}}$ which corresponds to rms value of the current. Draw a horizontal line which cuts the curve at $\frac{I_0}{\sqrt{2}}$. The difference betn $(f_2 - f_1)$ is known as band width.

Band width: It is the difference in the frequency corresponding to the rms value of current. $\Delta f = f_2 - f_1$.

Smaller the value of band width, sharper is the curve.

Half power points: The half power frequencies are the frequencies at which the power dissipation in the circuit drops to one half of its resonance value. $P = (I_0^2 \times R) = \left(\frac{E_0}{R}\right)^2 R$

power dissipation at f_1 is $I_1^2 R = \frac{1}{2}\left(\frac{E_0}{R}\right)^2 R$ $\quad I_1 = \frac{1}{\sqrt{2}}\frac{E_0}{R}$

Relation betn Band width and Quality factor :-

$$\frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{2}}\frac{E_0}{R}$$

At the resonance frequency ω_0 the impedence is R. So at ω_1 and ω_2 the impedence must be $\sqrt{2}\,R$.

ie $$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}\,R$$

squaring, $$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right) = \pm R$$

If $\omega_2 > \omega_1$ then

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \quad \text{—(1)}$$

$$\omega_2 L - \frac{1}{\omega_2 C} = R \quad \text{—(2)}$$

$$(\omega_1+\omega_2)L - \frac{(\omega_1+\omega_2)}{C\omega_1\omega_2} = 0$$

$$(\omega_1 + \omega_2)L = \frac{\omega_1+\omega_2}{C\omega_1\omega_2}$$

$$\therefore \omega_1\omega_2 = \frac{1}{LC}$$

Subtracting the eqn ① from ② we have

$$(\omega_2-\omega_1)L + \left(\frac{\omega_2-\omega_1}{\omega_1\omega_2}\right)\frac{1}{C} = 2R$$

ie $$(\omega_2-\omega_1)\left[L + \frac{1}{C\omega_1\omega_2}\right] = 2R$$

$$(\omega_2-\omega_1)\left[L+\frac{L\cancel{C}}{\cancel{C}}\right] = 2R$$

$$(\omega_2-\omega_1)\cancel{2}L = \cancel{2}R$$

$$\omega_2-\omega_1 = \frac{R}{L} \quad \text{—③}$$

It can be written as

$$\Delta\omega = \frac{R}{L} \quad \text{or} \quad \Delta f = \frac{R}{2\pi L}$$

$$\Delta f = \frac{R}{2\pi L}$$

The quality factor is given by

$$Q = \frac{\omega_0 L}{R} \quad \text{—④}$$

From ③ & ④ we have

$$\omega_2-\omega_1 = \frac{\omega_0}{Q} \quad \text{—⑤}$$

or $$\boxed{Q = \frac{\omega_0}{\omega_2-\omega_1}}$$ It gives relationship betn quality factor & band width.

From eqn ⑤ It is found that the bandwidth is inversely α^{al} to quality factor. As $\frac{\omega_0}{\omega_2-\omega_1}$ measures the sharpness of resonance, i.e. the quality factor directly measures the sharpness of resonance.

Sharpness of resonance :- It is the measure of rate of fall of amplitude from its maximum value at resonant frequency on either side of it. It is given by

$$\frac{f_0}{f_2 - f_1} \quad \text{or} \quad \frac{\omega_0}{\omega_2 - \omega_1}$$

Quality factor or Q factor :

The sharpness of resonance curve is determined by a quality factor called Q of the ckt.

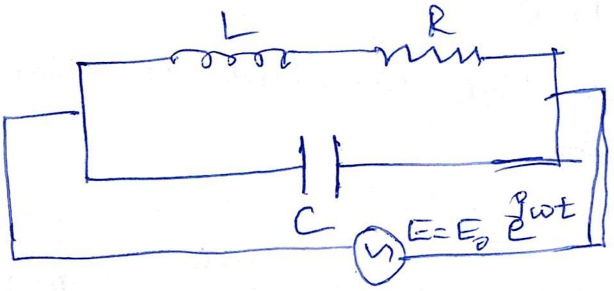
It is defined as the ratio of reactance of either the inductance or capacitance at the resonant frequency to the total resistance of the ckt.
Mathematically,

$$Q = \frac{X_L}{R} = \frac{\omega_0 L}{R}$$

$$Q = \frac{X_C}{R} = \frac{1}{\omega_0 C R}$$

LCR parallel circuit :-

The LCR ~~ckt~~ is said to be parallel, if the capacitor C is connected parallel to the series combination of R and L.



Let the applied voltage be

$$E = E_0 e^{j\omega t} \quad \text{--- (1)}$$

complex impedence of L branch, $Z_1 = R + j\omega L$ --- (2)

Complex impedence of C branch,

$$Z_2 = \frac{1}{j\omega C} \quad \text{--- (3)}$$

Z_1 and Z_2 are connected in parallel, so the total impedence of the ckt is given by $\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$

$$= \frac{1}{R + j\omega L} + \frac{1}{1/j\omega C}$$

$$= \frac{R - j\omega L}{(R + j\omega L)(R - j\omega L)} + j\omega C$$

$$= \frac{R - j\omega L}{R^2 + (\omega L)^2} + j\omega C$$

$$= \frac{R}{R^2 + (\omega L)^2} - \frac{j\omega L}{R^2 + (\omega L)^2} + j\omega C$$

$$= \frac{R}{R^2 + (\omega L)^2} + j\left(\omega C - \frac{\omega L}{R^2 + (\omega L)^2}\right)$$

Now $I = \frac{E}{Z} = E \times \frac{1}{Z}$

$$\therefore \quad I = E\left[\frac{R}{R^2 + (\omega L)^2} + j\left[\omega C - \frac{\omega L}{R^2 + (L\omega)^2}\right)\right]$$

Put $A \cos\phi = \frac{R}{R^2 + (\omega L)^2}$ --- (4)

$$A \sin\phi = \omega C - \frac{\omega L}{R^2 + (L\omega)^2} \quad \text{--- (5)}$$

ie $I = E\left[A\cos\phi + iA\sin\phi\right]$

$$= EAe^{i\phi}$$

$$I = E_0 A e^{i(\omega t + \phi)} \quad \text{--- (6)}$$

This gives the current in the parallel LCR circuit.

<u>phase difference</u>:

$\frac{(5)}{(4)}$ gives $\tan\phi = \dfrac{\omega C - \dfrac{\omega L}{R^2+(L\omega)^2}}{\dfrac{R}{R^2+(L\omega)^2}}$

or

$$\phi = \tan^{-1}\frac{\left(\omega C - \dfrac{\omega L}{R^2+(L\omega)^2}\right)}{\left(\dfrac{R}{R^2+(L\omega)^2}\right)}$$

Considering eqn. $Y = \frac{1}{Z}$ = admittance

ie $Y = \dfrac{R}{R^2+\omega^2L^2} + i\left(\omega C - \dfrac{\omega L}{R^2+(\omega L)^2}\right)$ --- (7)

The admittance is minimum or impedence is maximum at a particular frequency, when

$$\omega C = \frac{\omega L}{R^2+(\omega L)^2}$$

ie $C(R^2+\omega^2L^2) = L$

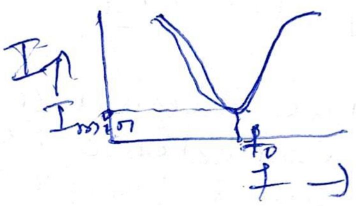
$$CR^2 + C\omega^2L^2 = L$$

$$C\omega^2L^2 = L - CR^2$$

$$\omega^2 = \frac{L - CR^2}{CL^2} = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \text{or} \quad \boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}} \quad \text{--- (8)}$$

At this frequency the admittance is minimum and hence the current is minimum. Such a frequency is called <u>Resonant frequency</u>. The variation of current with frequency is as shown in fig



<u>Q-factor</u> :- At resonant frequency, the value of admittance of the ckt will be

$$Y = \frac{R}{R^2 + \omega^2 L^2} \quad \text{or}$$

$$\text{Resistance}, Z = \frac{R^2 + \omega^2 L^2}{R}$$

Subs the value of ω, we get

$$Z = \frac{R^2 + L^2\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)}{R} = \frac{\cancel{R^2} + \frac{L}{C} - \cancel{R^2}}{R}$$

$$\boxed{Z = \frac{L}{RC}}$$

The current at resonance $= \frac{E_0}{Z} = \frac{E_0}{\frac{L}{RC}}$

i.e. current at resonance $= \frac{E_0 RC}{L}$

$$\therefore Q = \frac{\text{Amplitude of current across capacitance}}{\text{Amplitude of supplied current}}$$

$$= \frac{\cancel{E_0}\,\omega C}{\frac{\cancel{E_0} R C}{L}} = \frac{\omega L}{R}$$

It measures current amplification.

<u>Rejector circuit</u>: The parallel resonant ckt, rejects the current which has the natural frequency of the applied emf. But it allows the current of other frequencies. Hence the ckt is known as rejector ckt.

Comparison betn Series & Parallel ckt :-

Series Resonant ckt	Parallel resonant ckt
1) $f_0 = \frac{1}{2\pi\sqrt{LC}}$	1) $f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
2) It is so called acceptor circuit	2) Rejector ckt.
3) At the resonance, the impedence is minimum & equal to the resistance ~~of~~ the ckt.	3) At the resonance, the impedence is maximum & becomes ∞
4) It admits maximum current at resonant frequency.	4) It admits minimum current at the resonant frequency.
5) At the resonance it exhibits a voltage magnification equal to the quality factor	5) ~~At~~ At the resonance, it exhibits current magnification equal to the quality factor

ALTERNATING CURRENT

An alternating current is one which passes through a cycle of changes at regular intervals. Each cycle consists of two half cycles during one half of the cycle the current is entirely positive whereas the other half cycle the current is -ve.

The variation of emf is written as $E = E_0 \sin\omega t$

and the corresponding current is

$$I = I_0 \sin\omega t.$$

where E_0 is the peak value of voltage. E represents the voltage at any time 't', $\omega = 2\pi f = \frac{2\pi}{T}$ is the angular frequency of the voltage applied. The term ωt is called phase angle.

1) Cycle :- one complete set of +ve and -ve values of an alternating current or voltage is called a cycle.

2) Period (T) :- The time required to complete one cycle is called period. Its SI unit is second.

3) Frequency (f) :- The number of cycles performed by an alternating current for one second is called frequency. Its unit is hertz (Hz)

4) Phase :- The phase of an ac is the fraction of the time period of that alternating current which has elapsed since the current last passed through the zero position of the reference.

5) Peak value of Alternating current (I_0) :-

The maximum value of the current in either direction is called its peak value. It is denoted by I_0.

6) Average or mean value of alternating current (I_{av}) over one complete cycle is zero.

The value of current at any instant 't' is given by $I = I_0 \sin \omega t$

The average value of sinusoidal wave over one complete cycle is given by

$$I_{av} = \frac{\int_0^T I_0 \sin \omega t}{\int_0^T dt}$$

$$= -\frac{I_0}{\omega T} \cos \omega t \Big|_0^T$$

$$= -\frac{I_0}{\omega T}\left[\cos \frac{2\pi}{T}.T - \cos \frac{2\pi}{T}.0\right]$$

$$= 0$$

7) Average value of alternating current during half cycle :-

$I = I_0 \sin \omega t$

$$I_{avg} = \frac{\int_0^{T/2} I_0 \sin \omega t}{\int_0^{T/2} dt}$$

$$= \frac{2I_0}{T\omega}\left[-\cos \omega t \Big|_0^{T/2}\right.$$

$$= \frac{-\cancel{2}I_0}{\cancel{T}.\frac{2\pi}{\cancel{T}}}\left[\underset{(-1)}{\cos \frac{2\pi}{T}.\frac{T}{2}} - \underset{(1)}{\cos \frac{2\pi}{T}.0}\right]$$

$$= \frac{2I_0}{\pi} = \frac{2}{\pi}.I_0 = 0.637\ I_0$$

8) Root mean square or effective value of alternating current (I_{rms}) :-

It is defined as the square root of the mean of the squares of the instantaneous value over one complete cycle.

$$\bar{I}^2 = \frac{\int_0^T I_0^2 \sin^2 \omega t}{\int_0^T dt} \qquad \sin^2\theta = \left(\frac{1-\cos 2\theta}{2}\right)$$

$$= \frac{I_0^2}{2T}\int_0^T (1-\cos 2\omega t)\, dt$$

$$= \frac{I_0^2}{2T}\left[T - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{I_0^2}{2\cancel{T}} \times \cancel{T} = \frac{I_0^2}{2}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707 \times I_0$$

Thus the rms value of an ac is $\frac{1}{\sqrt{2}}$ times the peak value (I_0).

Complex Representation of AC ckt quantities

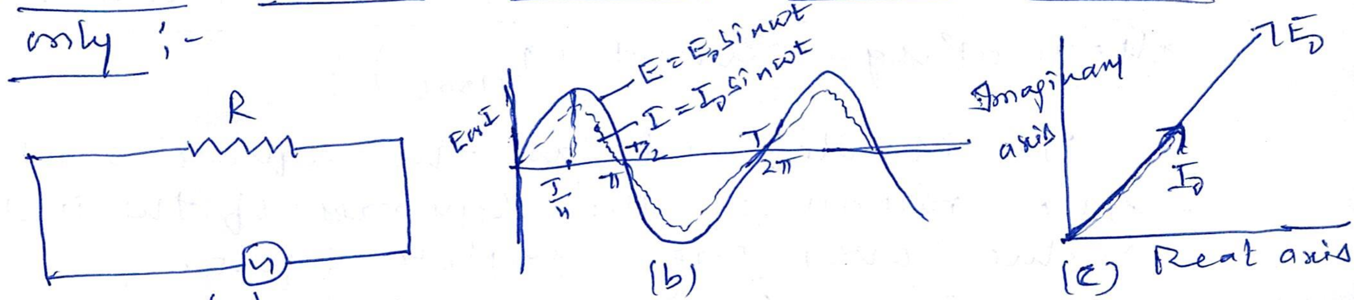
The expression of the type $z = x + jy$ is called complex number where x is called real part of complex number and y is the imaginary part of complex number and $j = \sqrt{-1}$

The sinusoidal voltage $E = E_0 \sin(\omega t + \alpha)$ and current $I = I_0 \sin(\omega t + \alpha)$ are nothing but the imaginary parts

ie $E = E_0 e^{j(\omega t+\alpha)}$ and

$I = I_0 e^{j(\omega t + \alpha)}$ respectively.

A.C. Circuit containing pure Resistance only :-



(a) (b) (c)

Consider an a.c ckt containing a pure Resistance R only. Let an alternating emf $E = E_0 \sin \omega t$ be applied to the circuit.

This emf is the imaginary part of the complex number.

$$\text{ie} \quad E = E_0 e^{j\omega t} \quad -(1)$$

The p.d developed across the resistance is IR where I is the instantaneous current in the ckt.

According to ohm's law, the pd across R is equal to the applied voltage

$$IR = E_0 e^{j\omega t}$$

$$I = \frac{E_0}{R} e^{j\omega t}$$

$$I = I_0 e^{j\omega t} \quad -(2)$$

where $I_0 = \frac{E_0}{R}$ maximum current in the ckt

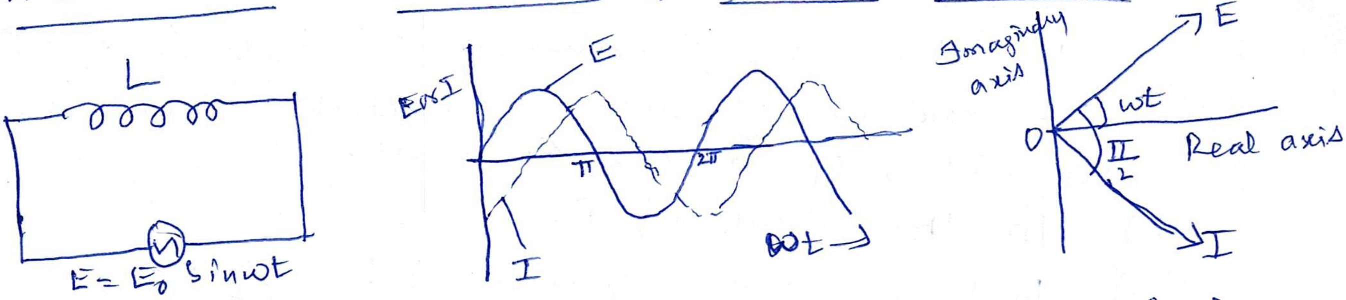
equns (1) & (2) shows that the current has the same form ~~and frequency~~ as the applied voltage. Thus the current is in phase with applied voltage as shown in fig(b).

Here R is the impedence of the resistance.

The inverse of impedence is called admittance.

$$\text{ie} \quad Y = \frac{1}{R}$$

AC Circuit containing pure Inductance :-



Consider an AC ckt containing a pure inductance L of zero resistance. Let an alternating emf $E = E_0 e^{j\omega t}$ (-1) applied to the circuit.

Let I be the instantaneous current in the ckt., $\frac{dI}{dt}$ be the rate of change of current at any instant. Induced emf $= -L\frac{dI}{dt}$

The total emf in the ckt is $E_0 e^{j\omega t} - L\frac{dI}{dt}$ & it is equal to zero as there is no resistance in the ckt.

$$E_0 e^{j\omega t} - L\frac{dI}{dt} = 0$$

$$dI = \frac{E_0}{L} e^{j\omega t}\, dt$$

on integration, we get

$$\int dI = \frac{E_0}{L}\int e^{j\omega t}\, dt$$

$$= \frac{E_0}{L}\frac{1}{j\omega} e^{j\omega t}$$

$$\frac{1}{j} = -j = e^{-j\frac{\pi}{2}} \quad \left[e^{-j\frac{\pi}{2}} = \cos\frac{\pi}{2} - j\sin\frac{\pi}{2} = 0 - j = -j \right.$$

$$I = \frac{E_0}{\omega L} e^{j\omega t} \cdot e^{-j\frac{\pi}{2}} = \frac{E_0}{\omega L} e^{j(\omega t - \frac{\pi}{2})}$$

$$I = I_0\, e^{j(\omega t - \frac{\pi}{2})} \quad \text{—(2)}$$

where $I_0 = \frac{E_0}{X_L}$ is the peak value of current

X_L is the inductive reactance

$$X_L = \omega L = 2\pi f L$$

It is the effective opposition offered by the inductor to the flow of AC & is defined as the ratio of rms value of voltage to the rms value of current thro' the inductor.

$$X_L = \frac{E_{rms}}{I_{rms}}$$

1) In the case of DC:

DC has no frequency so $f=0$ & $X_L=0$

It means inductor offers no opposition to the flow of direct current.

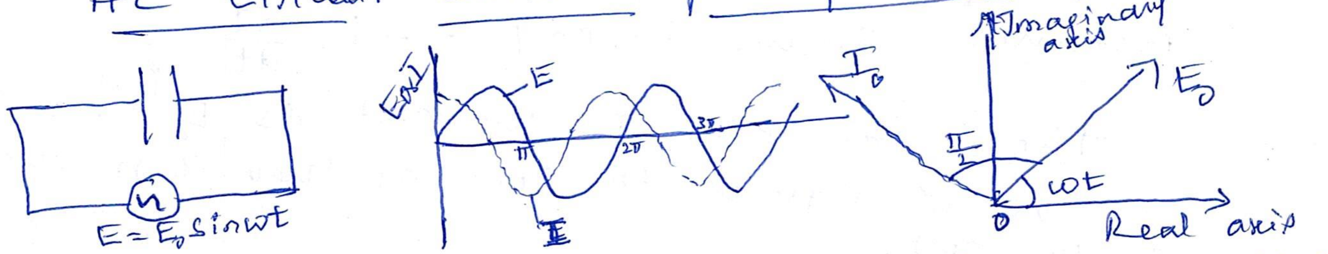
2) In the case of AC:-

For the given value of L, $X_L \propto f$

Thus as the frequency increases X_L also increases.

From eqns (1) & (2) The current lags behind the voltage by $\frac{\pi}{2}$.

AC circuit containing capacitance:



Consider an AC circuit containing a capacitor of capacitance C with an alternating voltage $E = E_0 \sin\omega t$. The imaginary part of the complex number

$$E = E_0 e^{j\omega t} \quad -(1)$$

Let q be the charge on the capacitor at any instant. pd developed across the capacitor is $\frac{q}{C}$

ie $\frac{q}{C} = E_0 e^{j\omega t} \Rightarrow q = C E_0 e^{j\omega t}$

diff this, $\frac{dq}{dt} = C E_0 \frac{d}{dt} e^{j\omega t}$

$$I = C E_0 j\omega e^{j\omega t}$$

$$I = C E_0 j\omega e^{j\omega t} = \frac{E_0}{\frac{1}{\omega C}} j e^{j\omega t}$$

$$= \frac{E_0}{1/j\omega C} e^{j\omega t}$$

But $e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2} = j$

Hence $I = \frac{E_0}{X_C} e^{j(\omega t + \frac{\pi}{2})}$

$$I = I_0 e^{j(\omega t + \frac{\pi}{2})} \quad -(2)$$

where $X_C = \frac{1}{\omega C}$ is capacitive reactance

The resonance is obtained when the current in the two branches is equal. Since the current is out of phase in 2 branches, the total current will be zero. Hence by eqn (1), at resonance.

$$\omega_0 C - \frac{1}{\omega_0 L} = 0$$

$$\omega_0^2 = \frac{1}{LC} \quad \boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

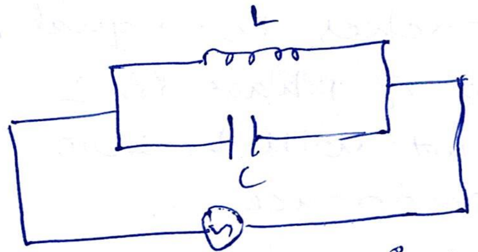
This is identical with ~~series~~ resonant ~~state~~ case.

When $I_T = 0$, $Z_T = \infty$

Thus at the frequency f_0 of the supply voltage, the ckt offers infinite impedence to the flow of current so that no current is drawn from the supply. Such a ~~current~~ ckt offering infinite impedence to the flow of current is called ||el resonant ckt & the particular frequency to is called resonant frequency.

Thus the current thro' the parallel resonant ckt is zero & it works as a perfect choke for AC & the ckt is rightly called the rejector ckt. Such ckts are used in radios as filter ckts.

AC ckt containing L and C :-



Consider an ac ckt containing an inductance L & a capacitance C connected in parallel. An alternating emf $E = E_0 e^{j\omega t}$ is applied across the combination.

The current thro' the inductance lags behind the applied emf in phase by $\frac{\pi}{2}$ is given by eqn

$$I_L = \frac{E_0}{j\omega L} e^{j\omega t} = \frac{E_0}{\omega L} e^{j(\omega t - \frac{\pi}{2})}$$

The current thro' the capacitor leads the applied emf in phase by $\frac{\pi}{2}$ & is given by $I_C = \frac{E_0}{\frac{1}{j\omega C}} e^{j\omega t} = \frac{E_0}{\frac{1}{\omega C}} e^{j(\omega t + \frac{\pi}{2})}$

∴ the current in the two branches differ in phase by π. Let I_C, I_L & I_T be the currents in capacitor, inductor & total current resply. Thus $I_T = I_L + I_C$

$$= \frac{E_0}{j\omega L} e^{j\omega t} + j E_0 \omega C e^{j\omega t}$$

$$I_T = j\left(\omega C - \frac{1}{\omega L}\right) E_0 e^{j\omega t} \quad \text{—— (1)}$$

The total impedence of the ||el ckt is given by $\frac{1}{Z_T} = \frac{1}{Z_L} + \frac{1}{Z_C} = \frac{Z_L + Z_C}{Z_L Z_C}$

$$Z_T = \frac{Z_L Z_C}{Z_L + Z_C} = \frac{j\omega L \cdot \frac{1}{j\omega C}}{j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$= \frac{\frac{L}{C}}{j\left(\omega L - \frac{1}{\omega C}\right)} \quad \times^{ly} \text{ \& } \div \text{ by } \frac{L}{L} \text{ \& } \frac{L}{C}$$

$$= \frac{\frac{L}{C}}{j\frac{L}{C}\left(\omega \cancel{L} \cdot \frac{C}{\cancel{L}} - \frac{1 \cdot \cancel{C}}{\omega \cancel{C} L}\right)} = \frac{\cancel{\frac{L}{C}}}{j\cancel{\frac{L}{C}}\left(\omega C - \frac{1}{\omega L}\right)}$$

$$Z_T = \frac{1}{j\left(\omega C - \frac{1}{\omega L}\right)} = \frac{E_0 e^{j\omega t}}{I_T}$$

Self Induction : The property of a coil by virtue of which any change in the current in the coil induces an emf in the coil, opposing the change is called self induction.

Coe of self induction :-

The magnetic flux ϕ is proportional to the current I.

i.e $\phi \propto I$ or $\phi = LI$

If $I = 1$ amp then $\phi = L$

Hence coefficient of self induction is numerically equal to the magnetic flux of the coil due to unit current in it.

or

$$e = -\frac{d\phi}{dt} = -\frac{d(LI)}{dt} = -L\frac{dI}{dt}$$

$$L = -\frac{e}{\frac{dI}{dt}}$$

L can also be defined as the opposing emf set up in it per unit rate of change of current in it.

The SI unit of L is Henry (H)

If $\frac{dI}{dt} = 1$ amp/s, $e = 1$ volt then $L = 1$ henry

The inductance of a coil has 1 henry if the induced emf in it is 1 volt when the current in the coil is changing at the rate of 1 ampere per second.

L of a coil depends upon

1) length of the coil
2) No of turns of the coil
3) cross sectional area of the coil
4) permeability of the medium inside the coil.

Power factor : may be defined as the ratio of true power to the apparent power.

1) power factor of LCR circuit

$$\cot\theta = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

2) Power factor for Inductive circuit

$$\cot\theta = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

3) Power factor for capacitive circuit

$$\cot\theta = \frac{R}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

4) Power factor for pure resistive circuit is 1

Then $\int_0^T \sin^2 \omega t \, dt = 0$

$\therefore$ Average power during one cycle

$$P = \frac{1}{2} E_0 I_0 \cos\phi$$

$$= \frac{E_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos\phi$$

$$= E_{rms} I_{rms} \cos\phi$$

Power in AC circuit containing LCR :-

The instantaneous values of voltage and current are given by

$$E = E_0 \sin \omega t$$

$$I = I_0 \sin(\omega t - \phi)$$

Instantaneous power $P = EI$

$$= E_0 \sin \omega t \; I_0 \sin(\omega t - \phi)$$

$$= E_0 I_0 \sin \omega t (\sin \omega t \cos\phi - \cos \omega t \sin\phi)$$

$$= E_0 I_0 \sin^2 \omega t \cos\phi - E_0 I_0 \sin \omega t \cos \omega t \sin\phi$$

$$= E_0 I_0 \sin^2 \omega t \cos\phi - \frac{1}{2} E_0 I_0 \sin 2\omega t \sin\phi$$

The average power during one complete cycle is given by

$$P = \frac{E_0 I_0 \cos\phi \int_0^T \sin^2 \omega t \, dt}{\int_0^T dt} - \frac{\frac{E_0 I_0}{2} \sin\phi \int_0^T \sin 2\omega t \, dt}{\int_0^T dt}$$

The average value of $\sin^2 \omega t$ for a complete cycle $= \frac{1}{2}$ & the average value of $\sin 2\omega t$ for a complete cycle $= 0$

$$\therefore P = E_0 I_0 \cos\phi \times \frac{1}{2} - 0$$

$$= \frac{E_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos\phi = E_{rms} I_{rms} \cos\phi$$

The term $E_{rms} I_{rms}$ is called apparent power & $\cos\phi$ is called power factor.

i.e. True power = Apparent power $\times \cos\phi$

Power in capacitive circuit:

The current in a capacitor circuit leads the applied emf in phase by $\frac{\pi}{2}$.

Thus $E = E_0 \sin\omega t$

$I = I_0 \sin(\omega t + \frac{\pi}{2})$

Instantaneous power $P = EI$

$= E_0 \sin\omega t \, I_0 \sin(\omega t + \frac{\pi}{2})$

$= E_0 I_0 \sin\omega t \cos\omega t$

$= \frac{1}{2} E_0 I_0 \sin 2\omega t$

∴ Average power over one cycle

is $= \frac{E_0 I_0}{2} \frac{\int_0^T \sin 2\omega t}{\int_0^T dt} = 0$

Thus the average power consumed in a pure capacitance circuit is zero & hence the current thro' it is also wattless.

Power in the AC circuit containing L and R:

Here emf leads the current by an angle ϕ. The instantaneous voltage, current & power will be.

$E = E_0 \sin\omega t$

$I = I_0 \sin(\omega t + \phi)$

$P = EI = E_0 I_0 \sin\omega t \cdot \sin(\omega t + \phi)$

$= E_0 I_0 \sin\omega t (\sin\omega t \cos\phi + \cos\omega t \sin\phi)$

$= E_0 I_0 (\sin^2\omega t \cos\phi + \sin\omega t \cos\omega t \sin\phi)$

$= E_0 I_0 \sin^2\omega t \cos\phi + \frac{1}{2} E_0 I_0 \sin 2\omega t \sin\phi$

For average power these two terms must be integrated betn. the ~~circuits~~ limits 0 to T for one complete cycle.

Power AC circuit containing pure Inductance:

In this circuit, current lags emf by $\frac{\pi}{2}$

The instantaneous voltage, current and power consumed are given by

$$E = E_0 \sin\omega t$$

$$I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

Instantaneous power $P = EI$

$$= E_0 I_0 \sin\omega t \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -E_0 I_0 \sin\omega t \sin\left(\frac{\pi}{2} - \omega t\right)$$

$$= -E_0 I_0 \sin\omega t \cos\omega t$$

$$= -\frac{1}{2} E_0 I_0 \sin 2\omega t$$

Average power for one complete cycle will be

$$P = -\frac{1}{2} E_0 I_0 \frac{\int_0^T \sin 2\omega t}{\int_0^T dt}$$

$$= 0$$

Thus power consumed in a pure inductance is zero & current in such circuit is known as wattless current. The choke coil works on this principle.

Power in AC circuits:

The power is the rate of doing work. In an electrical circuit, the power is defined as the rate at which the electrical energy is consumed in the circuit. As in a AC circuit both the applied emf and the current varying continously with time. Hence the power in an ac circuit is equal to the product of instantaneous emf and instantaneous current averaged over a complete cycle.

Purely resistive circuit

In the AC circuit containing resistance only, the voltage & the current are in the same phase. Instantaneous values of voltage & the current are

$$E = E_0 \sin\omega t \quad , I = I_0 \sin\omega t$$

Instantaneous power $= EI$

$$= E_0 I_0 \sin^2\omega t$$

$$= E_0 I_0 \left(\frac{1-\cos 2\omega t}{2}\right)$$

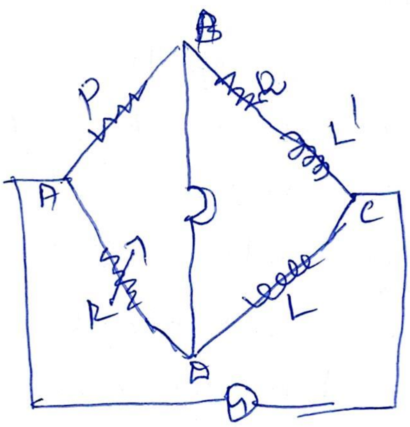
$$= \frac{1}{2} E_0 I_0 - \frac{1}{2} E_0 I_0 \cos 2\omega t$$

Average power for one complete cycle is

$$P = \frac{\frac{1}{2} E_0 I_0 \int_0^T dt}{\int_0^T dt} - \frac{\frac{1}{2} E_0 I_0 \int_0^T \cos 2\omega t \, dt}{\int_0^T dt}$$

$$= \frac{1}{2} E_0 I_0 - 0 = \frac{E_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}}$$

Real power = Apparent power.



Maxwell's Bridge :-

It is used ~~to~~ measure the value of coefficient of self inductance of a coil.

P Q R are the non-inductive resistances. L' and L are the coils connected as shown in circuit diagram. We know that

$P = Z_1$, $Q + L' = Z_2$, $R = Z_3$ & $Z_4 = L$

But $\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$

ie $\frac{P}{Q+L'} = \frac{R}{L}$

ie $\frac{P}{Q+j\omega L'} = \frac{R}{j\omega L}$

$$\frac{Q+j\omega L'}{P} = \frac{j\omega L}{R}$$

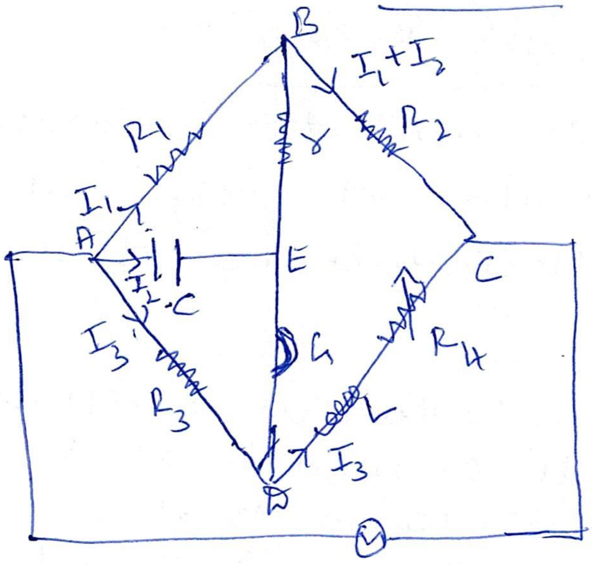
$$\frac{Q}{P} + j\frac{\omega L'}{P} = j\frac{\omega L}{R}$$

equating the coefficients of imaginary parts we get

$$\frac{L'}{P} = \frac{L}{R} \Rightarrow \boxed{L = L'\frac{R}{P}}$$

Procedure: The resistances P & Q are fixed along with the known value of L'. The value of R is varied until the sound in the head phone becomes minimum. Under this condition the potential at the points B and D are equal in magnitude. The value of L can be determined by knowing L', R and P respectively.

Andersons bridge :—



This is one of the most accurate bridge which is used for measuring self inductance.

It is modification of Maxwell's bridge. In this bridge double balance is obtained by the variation of resistance alone. The value of capacitance being kept constant. The circuit diagram of bridge is as shown in fig. The coil whose self inductance L is to be determined is connected for the arm CD along with a variable non-inductive resistance. Arms AB, BC & AD contain non-inductive resistance R_1, R_2 & R_3 respectively. Resistance r is connected in series with standard capacitor C of fixed value and this combination is putting parallel with arm AB. One end of the head phone is connected to the junction of C & r while the other to D.

Let the current distribution for the bridge is as shown in figure. Applying Kirchoff's voltage law to the meshes ABEA, AEDA and BCDB,

we get, from ABEA,

$$I_1 R_1 - I_2 r - I_2 \frac{1}{j\omega C} = 0$$

$$I_1 R_1 = I_2 r + I_2 \frac{1}{j\omega C}$$

$$I_1 R_1 = I_2 \left(r + \frac{1}{j\omega C} \right)$$

$$I_1 = \frac{I_2}{R_1} \left(r + \frac{1}{j\omega C} \right) \quad \text{—— (1)}$$

From mesh AE~~D~~A,

$$\frac{I_2}{j\omega C} - I_3 R_3 = 0$$

$$I_3 = \frac{I_2}{j\omega C \times R_3} \quad \text{--- (2)}$$

For mesh BCDB

$$(I_1 + I_2) R_2 - I_3 (R_4 + j\omega L) + I_2 r = 0$$

$$I_1 R_2 + I_2 R_2 - I_3 R_4 - I_3 j\omega L + I_2 r = 0$$

$$I_1 R_2 + I_2 (r + R_2) - I_3 (R_4 + j\omega L) = 0 \quad \text{--- (3)}$$

Subs the value of I_1 & I_3 from eqn. (1) & (2) in eqn. (3) we get

$$\frac{I_2 R_2}{R_1}\left(r + \frac{1}{j\omega C}\right) + I_2 (r + R_2) - \frac{I_2}{j\omega C R_3}(R_4 + j\omega L) = 0$$

$$\frac{R_2}{R_1}\left(r + \frac{1}{j\omega C}\right) + (r + R_2) - \frac{1}{j\omega C R_3}(R_4 + j\omega L) = 0 \quad \text{--- (4)}$$

Equating real and imaginary parts of eqn. (4) seperately we get

$$\frac{R_2}{R_1} r + r + R_2 - \frac{L}{C R_3} = 0 \quad \text{--- (5)}$$

$$\frac{R_2}{R_1 \omega C} = \frac{R_4}{R_3 \omega C} \quad \text{--- (6)}$$

from (5) we have

$$\frac{L}{C R_3} = \frac{R_2}{R_1} r + r + R_2$$

$$= \left(\frac{R_2}{R_1} + 1\right) r + R_2$$

$$L = R_3 C \left[\left(\frac{R_2}{R_1} + 1\right) r + R_2\right]$$

$$L = C \left[\left(\frac{R_2 R_3}{R_1} + R_3\right) r + R_2 R_3\right] \quad \text{--- (7)}$$

From eqn (6) we have

$$\frac{R_2}{R_1} = \frac{R_4}{R_3}$$

$$R_4 = \frac{R_2 R_3}{R_1} \quad \text{--- (8)}$$

Subst the value of R_4 in eqn (7)

we get

$$L = C\left[(R_4 + R_3)r + R_2 R_3\right] \quad \text{--- (9)}$$

eqn (9) is used to find the value of unknown inductance L.

eqn (8) gives DC balance

AC balance is done by adjusting r.